A Model of Vehicle Routing Problem Minimizing Energy Consumption in Urban Environment

Weizhen Rao, Chun Jin

1 Institute of Systems Engineering, Dalian University of Technology, Dalian 116024, China
E-mail: raoweizhen@mail.dlut.edu.cn

2 Institute of Systems Engineering, Dalian University of Technology, Dalian 116024, China
E-mail: jinchun@dlut.edu.cn

Abstract

The energy consumption of vehicles in urban distribution is worthy of attention for both low-carbon logistics and deduction of distribution cost. The classical models of logistics distribution generally focused on minimizing the distance covered by vehicles and paid little attention to the properties of urban distribution. This paper presents a model of urban vehicle routing problem with the object function minimizing the energy consumption of vehicles (VRPMEC) based on the classical capacitated vehicle routing problem (CVRP). By analyzing properties of urban distribution, we found that VRPMEC is more difficultly solved than CVRP by comparing their solution space and the complexity of implementing 2-opt, or-opt, exchange and swap in their solutions. However, the energy consumption of vehicles and distance covered by vehicles are significantly correlated in the same solution. A multi-objective optimization model for VRPMEC and its solving strategy were proposed. The experiment on the instances extended based on benchmark CVRP instances has carried out and the results have indicated that the proposed strategy is effective and efficient.

Keywords: Urban distribution; Vehicle routing problem; Energy consumption; Multi-objective optimization model

1 Introduction

The Vehicle Routing Problem (VRP) is a classic combinatorial optimization problem that has extensively studied in operations research field since its introduction by Dantzig and Ramser[Dantzig and Ramser, 1959]. By considering additional requirements and various constraints on route construction, different vehicle routing problems (VRPs) have been formulated. The majority of these VRPs aim to minimize the length covered by vehicles, the number of vehicles dispatched, the total time and so on. In fact, the final goal of VRPs is to minimize the total cost. So the sum of energy consumed by all vehicles dispatched is the greater concern to companies that pursue fuel cost savings.

The amount of energy consumed by a vehicle depends on load, distance, terrain and speed, the sum of arrival times at customers[Ngueveu et al, 2009], and other factors[Hellstrom et al, 2009; Tavares et al, 2009; Carvalho et al, 2012]. A wide range of models have been proposed to predict fuel consumption and emission rates such as CVRP considering the fuel consumption rate[Xiao et al, 2012], the pollution-routing problem model [Bektas and Laporte, 2011]. However, from the perspective of minimizing energy consumption in transportation planning, until now a little research has been carried out. Kara[Kara et al, 2007] proposed a energy minimizing vehicle routing problem in which only considered two factors of load weight and distance discovered by vehicle. The Ph.D dissertation of Palmer[Palmer, 2007] presented an integrated routing and emissions model for freight vehicles and investigates the role of speed in reducing CO2 emissions under various congestion scenarios and time window settings. However, Palmer[Palmer, 2007] did not account for vehicle loads in his model, although this was offered as a future research topic. Maden [Maden et al, 2010] considered a vehicle routing and scheduling problem with time windows in which speed depends on the time of travel. Fagerholt[Fagerholt et al, 2010] proposed an alternative solution methodology in which the arrival times was divided and the problem was solved as a shortest path problem on a directed acyclic graph. Carvalho[Carvalho et al, 2012]
identified energy waste streams in vehicles fuel consumption and introduced the concept of lean driving systems. Hu[Hu et al, 2007] presented an intelligent solution system for a VRP with rigid time window in urban distribution. Cappiello[Cappiello et al, 2002] developed and implemented an instantaneous statistical model of emissions (CO₂, CO, HC, and NOₓ) and fuel consumption for light-duty vehicles, which is simplified from the physical load-based approaches that are gaining in popularity.

This brief survey shows that there is a gap in the literature on the application of energy consumption based models in VRPs. Indeed, most studies fail to properly integrate load, distance, terrain and speed factors.

In this paper, we aim to present a model of vehicle routing problem minimizing the energy consumption (VRPMEC). In particular, our goal is to: (i) describe an approach to reduce energy requirements of vehicle routing based on a comprehensive emissions model that takes into account a number of factors including load, distance, terrain and speed; (ii) analyze and compare the complexity of implementing four operators (2-opt, or-opt, exchange and swap) in VRPMEC solution with that of classic CVRP solution; and (iii) propose a solving strategy for VRPMEC that can be combined with all algorithms solving the classic CVRP in order to solve VRPMEC efficiently and effectively.

The remainder of this paper is organized as follows. Section 2 proposes the VRPMEC model and presents notations used in this paper. The complexity of implementing four operators in VRPMEC and CVRP solution is analyzed and compared in Section 3. In Section 4, a search strategy with multi-objective optimization model is proposed. The experiments are carried out and analyzed in Section 5. Finally, Section 6 contains the conclusions of this paper.

2 Formulation of VRPMEC

2.1 A Comprehensive Emissions Model

Up to present, there have been several emissions models proposed by researchers in transportation area. Typically, in 2009, Barth[Barth and Boriboonsomsin, 2009] developed a classical emission model based on formula presented by Ross in 1994. The model is considered as a constant.

$$ FR = \frac{\varphi (kND + P / \eta)}{\mu} \quad (1) $$

where,

- $\varphi$- the fuel/air equivalence ratio;
- $k$-the engine friction factor;
- $N$-the engine speed;
- $D$-the engine displacement;
- $P$-the power requirements on the engine;
- $P_{\text{acc}}$-tractive power requirements;
- $M$-the mass (kg) of the vehicle (empty plus carried load);
- $a$-the acceleration (m/s²);
- $v$-the vehicle velocity (meters/second);
- $g$-the gravitational constant (9.81 m/s²);
- $\theta$-the road angle;
- $A$-the frontal surface area of the vehicle (m²);
- $\rho$- the air density (kg/m³);
- $C_d$- the coefficients of rolling resistance;
- $C_{f, r}$ the coefficients of rolling drag;
- $P_{\text{acc}}$-the engine power demand associated with the operation of accessories, such as air conditioning, power steering and brakes, and electrical loads;
- $\eta$-a measure of indicated engine efficiency;
- $\eta_{f, r}$-the combined efficiency of the transmission and final drive;
- $\mu$-a parameter depends on some constants including $N$.

Apparently, all factors described above have impact on emissions/energy consumption in some extent. It should be pointed out that FR in formula (1) is the amount of energy consumption per unit time. Then the energy consumption is related with the driving time of vehicle that is right the distance $d$ discovered by vehicles if vehicle velocity be considered as a constant.

2.2 The Model of Work Done by Vehicle

It has been recognized that the energy consumption of a vehicle traveling along a route depends on many factors as described above. These can be divided into two sets. Factors in the first set have a direct relationship with the traveling schedule. The factors in the second set have no direct relationship with the traveling schedule. The detailed division of all factors in formula (1) is shown in Table 1.

In other words, factors in the first set are independent of the traveling schedule. However, factors in the second set are closely related with the traveling schedule. Then we propose a model of energy consumption based on factors in the second set.

In the respect of the work done by vehicle, we analyze the energy consumption of vehicle with uniformly velocity on upgrade/downgrade road. Other factors hypothesized to same, the more the work is done by vehicle, and the more the energy consumption is consumed by the vehicle. The force analytical graph of vehicle with uniformly velocity on upgrade and downgrade road are illustrated in Figures 1 and 2, respectively. The traction force $F$ in the two cases can
be computed as formulas (2) and (3) according to the force balance theorem.

\[ F = Mg(\cos \alpha C_r + \sin \theta) \]  
(2)

\[ F = \begin{cases} 
Mg(\cos \alpha C_r - \sin \theta) & \text{if } \cos \alpha C_r > \sin \theta \\
0 & \text{otherwise}
\end{cases} \]  
(3)

where \( F \) equal to 0 in formula (3) denotes that the down grade road angle is enough big and vehicle has to apply the brake to keep the uniformly velocity.

\[ F = \frac{Mg\sin \theta}{\cos \theta} \]  

Formula (5) is used to calculate the work done by a vehicle, where \( d \) is the distance discovered by the vehicle.

\[ W = F \cdot d \]  
(5)

Apparently, as the coefficients of rolling resistance \( C_r \) become bigger, \( F \) and \( W \) become bigger accordingly. According to the study of some researcher, \( C_r \) is closely related the vehicle velocity \( v \).

\[ F = \frac{Mg}{v^3} \]  

\[ W = \frac{Mg}{v^2} \]  

\[ \alpha = \begin{cases} 
\theta & \text{upgrade road} \\
-\theta & \text{downgrade road}
\end{cases} \]  

\[ = \begin{cases} 
\frac{Mg\omega \cdot d}{\cos \alpha} & \omega > 0 \\
0 & \text{otherwise}
\end{cases} \]  
(7)

\[ W = \frac{Mg\omega \cdot d}{\cos \alpha} + \sin \alpha \]  

\[ \alpha = \begin{cases} 
\theta & \text{upgrade road} \\
-\theta & \text{downgrade road}
\end{cases} \]  

\[ \omega = \begin{cases} 
\text{the vehicle velocity (meters/second);} & \theta \text{ the road angle;}
\theta \text{ the road angle;}
\end{cases} \]  

\[ \omega = \begin{cases} 
\eta C_r & v \in [l, u], u \leq v_{\text{max}} \\
0 & \text{otherwise}
\end{cases} \]  
(6)

where, \( l \) and \( u \) are the lowest and largest velocity of vehicle, \( v_{\text{max}} \) approximately equal to 50-60km/h. Then the work done by a vehicle on gradient road can be computed as formula (7).

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0 & \text{otherwise}
\end{cases} \]  
(6)
customers. The positions of all nodes are denoted as a coordination \((x_i, y_i, z_i), 0 \leq i \leq n\). If \(z_i > z_j\), that means road from customer \(i\) to customer \(j\) is downgrade, and vice versa.

Some notations in VRPMEC:
- \(d_{ij}\) the Euclidean distance from nodes \(i\) to \(j\);
- \(c_{ij}\) the Euclidean plane distance from nodes \(i\) to \(j\);
- \(Q_{ij}\) the carried load (kg) of the vehicle from nodes \(i\) to \(j\);
- \(v_{ij}\) the uniformly velocity of the vehicle from nodes \(i\) to \(j\);
- \(x_{ij}\) the distance from customers \(i\) to \(j\) with uniformly velocity \(v_{ij}\).

The optimization model of VRPMEC is formulated as follows.

**Objective function (8)** minimizes the sum of distance from customers.

$$\sum_{i \neq j \in A} w_{ij} x_{ij}$$

$$w_{ij} = \begin{cases} 0 & \text{if } x_{ij} = 0 \\ \frac{c_{ij} \cdot v_{\max}}{v_{ij}} + (z_j - z_i) & \text{if } x_{ij} = 1 \\ \frac{c_{ij} \cdot v_{\max}}{v_{ij}} + (z_j - z_i) - \frac{d_{ji}}{d_{ij}} \cdot \max & \text{otherwise} \end{cases}$$

**Subject to**

\(\sum_{i \neq j \in A} x_{ij} = k\) \hspace{1cm} (9)

\(\sum_{i \in V_0} x_{ij} = 1 \hspace{1cm} \forall i \in V_0\) \hspace{1cm} (10)

\(\sum_{j \in V_0} x_{ij} = 1 \hspace{1cm} \forall j \in V_0\) \hspace{1cm} (11)

\(q_i x_{ij} \leq Q_{ij} \leq (Q - q_i)x_j \hspace{1cm} \forall (i, j) \in A\) \hspace{1cm} (12)

\(\sum_{j \in V_0} (Q_{ij} - Q_{ji}) = q_i \hspace{1cm} \forall i \in V_0\) \hspace{1cm} (13)

\(Q_{ij} \geq 0 \hspace{1cm} \forall (i, j) \in A\) \hspace{1cm} (14)

\(x_{ij} \in \{0, 1\} \hspace{1cm} \forall (i, j) \in A\) \hspace{1cm} (15)

Objective function (8) minimizes the sum of energy consumption of all vehicles. Constraint (9) denotes that dispatch \(k\) vehicles and \(k\) may be variable. Constraints (10) and (11) represent that each customer must be visited and can only be visited by one vehicle. Constraint (12) represents that any \(Q_{ij}\) can not exceed carried load capacity of vehicle \(Q\). Constraints (13) indicate that the reduced cargo of the vehicle after it visits a customer is equal to the demand of the customer. Constraint (14) limits \(Q_{ij}\) not less than zero. integrality constraints are required in Constraint (15).

It should be pointed out that VRPMEC is the generalization of the classic CVRP when \(r\) is far greater than 1 and any \(v_{ij} = v\). That is because the objective function can be rewritten as formula (16) for \(r\) is far greater than 1 and \(v_{ij} = v\).

$$w_{ij} = \frac{rQ + Q_{ij}}{v} \left(\frac{c_{ij} \cdot v_{\max}}{v_{ij}} + \frac{(z_j - z_i)}{d_{ij}}\right)$$

$$= \beta \cdot c_{ij}$$

Formula (16) means that \(w_{ij}\) is proportional to \(c_{ij}\). In other words, the objective function of VRPMEC is proportional to the objective function of CVRP. So CVRP is the special case of VRPMEC, in which \(r\) is far greater than 1 and \(v_{ij} = v\).

In this paper, it is assumed that the velocity \(v_{ij}\) in urban can be calculated as formula (17).

$$v_{ij} = v_\text{min} + \frac{ud_i + ud_j - (v_{\max} - v_{\min})}{2ud_i}$$

where \(v_{\max}\) and \(v_{\min}\) are the largest and lowest velocity of vehicle in urban, respectively. \(ud_i\) and \(ud_j\) denote distance from customers \(i\) and \(j\) to city center.

## 3 Complexity of solving VRPMEC

### 3.1 The Solution Space of VRPMEC

The solution space is the set of all feasible solutions. Apparently, the more the feasible solutions of one problem have, the more difficulty solving the problem is. In order to analyze the difficulty solving VRPMEC, it is necessary to compare the solution space of VRPMEC with that of CVRP. As illustrated in VRPMEC, CVRP is the special case of VRPMEC. Then any feasible solution in solution space of CVRP is feasible solution in solution space of VRPMEC too. The key difference between solutions of VRPMEC and of CVRP is that the former has direction. To speak exactly, a feasible solution of CVRP with only one vehicle corresponds to two different solutions of
VRPMEC (as shown in Figure 3).

However, the solution of VRPMEC and CVRP include more than one vehicle generally. It is assumed that $\Omega_c$ denotes the solution space of CVRP and $\Omega_V$ denotes the solution space of VRPMEC. And notations $|\Omega_c|$ and $|\Omega_V|$ represent the number of all feasible solutions in $\Omega_c$ and $\Omega_V$, respectively. Any feasible solution in $\Omega_c$ and $\Omega_V$ limit every vehicle dispatched can not be overloaded. Let $k_{\min}$ represent the minimal number of vehicle dispatched when $n$ customers with demand must be serviced. Then $k_{\min}$ can be computed as formula (18) stated as follows.

$$k_{\min} = \frac{\sum_{i=1}^{n} q_i}{Q}$$  \hspace{1cm} (18)

As illustrated in Figure 3, the feasible solutions in $\Omega_c$ have direction and those in $\Omega_V$ have no direction. So a feasible solution with $k$ vehicles in $\Omega_c$ will correspond to $2^k$ feasible solutions in $\Omega_V$. Apparently, the inequality (19) is constantly held.

$$|\Omega_V| \geq 2^k - |\Omega_c|$$  \hspace{1cm} (19)

From inequality (19), it is known that the feasible solutions in solution space $\Omega_V$ are far more than those in solution space $\Omega_c$. This means that the difficulty of searching a good solution in $\Omega_V$ is much more than that in $\Omega_c$.

### 3.2 The Complexity of Implementing Four Operators in VRPMEC Solution

In general, three steps are implemented with the algorithms to solve VRP. The three steps are described as follows.

- **Step 1:** generate initial solutions by using construction algorithms;
- **Step 2:** based on the initial solution, develop some rules and continually improve the current best solution and record the best solution;
- **Step 3:** judge whether termination conditions of the algorithm is satisfied or not. If it is, then output the current best solution; otherwise, go step2.

Actually, the main part of consuming time is the process of implementation of step 2. It is essential that whether the implementation of step 2 is efficient. In step 2, there are four operators: 2-opt, or-opt, exchange and swap, usually included in algorithms solving VRP. In order to show the difficulty in solving VRPMEC, we analyze the complexity of implementing the four operators in solution by VRPMEC.

Figure 4 shows the implementation of the operators 2-opt and or-opt in solution by VRPMEC, where operator 2-opt is shown in Figure 4(a) and or-opt in Figure 4(b). And Figure 5 shows the implementation of the operators exchange and swap in solution by VRPMEC, where operator exchange is shown in Figure 5(a) and swap in Figure 5(b).
It can be clearly seen that 2-opt, or-opt are intra-operators that only change the sequence of customers serviced by one vehicle, and exchange, swap are inter-operators that change the number and sequence of customers serviced by two vehicles at least.

The essential rule of implementing the four operators in an algorithm is that whether the objective function is better or not after implementation of the four operators. The objective function of VRPMEC is shown in formula (8) that is related with $f_{ij}$ and $Q_{ji}$. Because $Q_{ji}$ is the carried load of the vehicle from customers $i$ to $j$, many $Q_{ji}$ in solution have to be updated after implementing the any one of four operators. For example, all $Q_{ji}$ from customers $j$ to $j+p$ will be updated after implementation of or-opt shown in Figure 4(b).

However, the objective function of CVRP is to minimize the total distance discovered by all vehicles dispatched. It is easily determined that whether the solution after implementing one of the four operators is improved or not by computing the difference between the sum of length of added edges and the sum of length of deleted edges. For example, because the sum of length of added edges $d_{i,j} + d_{j,i+1}$ is less than the sum of length of deleted edges $d_{i,j} + d_{j,i+1}$, it can be concluded that the solution of CVRP is improved by implementing 2-opt as shown in Figure 4(a).

From the above, comparing with solution in CVRP, it is more complex to evaluate that whether a solution in VRPMEC is improved or not. In order to scientific illustrate the point, we analyze the complexity of implementing the four operators in VRPMEC and CVRP, as shown in formulas (20)-(27).

\[
\text{Complexity}_{i\rightarrow j}^{\text{opt}} = O(j + 1 - i - 1) = O(j + p - i) = O(p) \leq O(n) \leq O(n) \quad (20)
\]

\[
\text{Complexity}_{i\rightarrow j}^{\text{or-opt}} = O(t - i) = O(p + 3) \quad (21)
\]

\[
\text{Complexity}_{i\rightarrow j}^{\text{or-opt}} = O(b) = O(d) \quad (22)
\]

\[
\text{Complexity}_{i\rightarrow j}^{\text{exchange}} = O(b) = O(d) \quad (23)
\]

From formulas (20)-(23), the complexity of implementing four operators in solution of VRPMEC is $O(n)$; and from formulas (24)-(27), the same index is $O(1)$ in CVRP. Apparently, the difficulty of solving VRPMEC is far more than solving CVRP.

4 A Strategy Solving VRPMEC

4.1 Comparison of Solutions in VRPMEC with CVRP

Due to VRPMEC is a new variant of VRP, so there are not the benchmark instances of VRPMEC. In order to compare the optimal solution of VRPMEC with that of CVRP, we devise an instance of VRPMEC with 6 customers. The information of instance and the values of related parameters are shown in Table 2.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>$x$ (km)</th>
<th>$y$ (km)</th>
<th>$z$ (km)</th>
<th>$q$ (kg)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>-</td>
<td>$r=1$</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>12</td>
<td>0.05</td>
<td>320</td>
<td>$Q=1000$ kg</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>16</td>
<td>0.06</td>
<td>250</td>
<td>$v_{\text{max}}=60$ km/h</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>5</td>
<td>0.01</td>
<td>60</td>
<td>$v_{\text{min}}=20$ km/h</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>8</td>
<td>0.03</td>
<td>180</td>
<td>$u_{d}=20$ km</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2</td>
<td>0.02</td>
<td>150</td>
<td>$C=0.01$</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>6</td>
<td>0.03</td>
<td>100</td>
<td>$g=9.81$</td>
</tr>
<tr>
<td>Center 5 &amp; 5</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This VRPMEC instance has 1800 feasible solutions. The objective functions of VRPMEC and CVRP are computed based on each solution. The computational results are illustrated in Figure 6. Furthermore, the best solutions of VRPMEC and CVRP are generated, and their total distance and the energy consumption are compared to find the difference between energy minimizing and distance-minimizing solutions. The best solutions of VRPMEC and CVRP are described in Figures 7 and 8, and their numerical results are shown in Table 3.
Corresponds to two feasible solutions of VRPMEC, which

Objective Function of CVRP

Table 3

As Table 3 demonstrates, the best solution of CVRP corresponds to two feasible solutions of VRPMEC, which

their energy consumptions are 13883.46 and 14036.32, respectively. However, the optimal solution of VRPMEC has energy consumption 13554.14. In other words, there is a considerable difference between energy consumption minimizing and distance-minimizing solutions. Although the difference is only about 3%, this difference may play the essential role of reducing the logistics cost in real word.

It can be drawn from Figure 6 that in general, there is a significantly positive correlation between the two objective functions. By calculating, the correlation coefficient is about 0.88. This means that a good solution of CVRP may correspond to a not bad (just good) solution of VRPMEC.

4.2 The Multi-objective Functions Strategy

From analysis above, VRPMEC is a far more difficulty problem than the traditional CVRP. In addition, the good solutions in CVRP may be the good solutions in VRPMEC too. Thus we propose a strategy to improve the efficiency of algorithms solving VRPMEC.

The strategy named two objective functions strategy (TOFS) that can be stated as follows. We divide the process of solving VRPMEC into two stages. In the first stage, the aim is to find a good solution of CVRP at minimizing total distance. In the second stage, the algorithm focuses on minimizing the energy consumption to get a good solution of VRPMEC based on the good solution of CVRP generated in the first stage. The main procedure of a local search algorithm with TOFS is shown in Figure 9.
5 Experimental Results

In this section, a numerical experiment is carried out to verify our proposed strategy. Firstly, a VRPMEC instance named Golden-200-5 is devised based on a CVRP standard instance Golden-200-5 (denotes with 200 customers and 5 vehicles). Then, instance Golden-200-5 is solved by using a local search algorithm (LSA) with (LSA+TOFS) or without TOFS in order to evaluate the efficiency and effectiveness of TOFS.

LSA algorithm is coded with MATLAB7.9 and performed on a computer equipped with CPU Inter(R) core (TM) Q9400, main frequency 2.66Ghz, memory 4.0G, and operation system WINDOWS XP.

The values of parameters in VRPMEC model are the same as those shown in Table 2, and the value z of 200 customers and depot is presented in Table 4.

The computational results from instance Golden-200-5 solved by LSA and LSA+TOFS are demonstrated in Table 5. It should be stated that the objective function is the amount of energy consumption in VRPMEC, and is the distance in CVRP. From Table 5, it is known that the solution generated by LSA without TOFS is about 0.05% better than that by LSA+TOFS. However, the total running time of LSA is about 4 times as fast as that of LSA+TOFS. Thus TOFS can significantly improve the efficiency of algorithm at cost of the tiny quality of solution.

Table 5 The computational results of the Golden-200-5 instance solved by LSA and LSA+TOFS

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>VRPMEC</th>
<th>CVRP</th>
<th>Stage time(s)</th>
<th>Total time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSA stage 1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.36</td>
</tr>
<tr>
<td>LSA stage 2</td>
<td>13329282.80</td>
<td>6764.47</td>
<td>2.36</td>
<td>-</td>
</tr>
<tr>
<td>LSA+TOFS stage 1</td>
<td>1372367.31</td>
<td>6650.23</td>
<td>0.15</td>
<td>0.53</td>
</tr>
<tr>
<td>LSA+TOFS stage 2</td>
<td>1333640.61</td>
<td>6670.08</td>
<td>0.38</td>
<td>0.53</td>
</tr>
</tbody>
</table>

6 Conclusions

The vehicle routing problems have been studied for more than fifty years. However, these VRPs aim at minimizing the total distance, the number of vehicles dispatched, or total time and so on. In the respective of deduction of energy consumption, this paper proposes a new objective function model for the vehicle routing problem (VRPMEC) in which the load weight, speed, terrain and distance factors are included. To more exactly denote the distribution cost in real word.

By analyzing the solution space of VRPMEC and complexity of implementing four operators in solution of VRPMEC, it is found that the VRPMEC is far more difficultly solved than the classical CVRP. Thus we propose a strategy solving VRPMEC (TOFS) that can be used by combining with any exiting algorithm of CVRP. The experimental results indicate that a local search algorithm with TOFS is far better efficient and slightly decrease the quality than traditional local search algorithm.

In the future study, we may consider more factors affecting energy consumption of vehicle, such as traffic jam, different types of vehicle to develop a more real-situation model.

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References


